Sustainability of public finances

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Contents

- In the 1980s, 1990s, and 2000s several western countries faced increasing budgetary imbalances.

- In the European Union, the sustainability of public finances is a key feature in the Maastricht Treaty.

- To adhere to the Economic and Monetary Union countries need a budget deficit below 3% of GDP and a debt ratio below 60%.

- This fiscal framework is present in the Stability and Growth Pact, which strives for sustainable and sound fiscal positions, a necessary condition for the implementation of a single monetary policy.

- Several studies have addressed the sustainability of public finances, assessing whether the intertemporal government budget constraint is fulfilled (see Afonso, 2005, Afonso, Rault, 2010, for an overview).

- Such assessments test empirically

  - for the cointegration between government revenue and government spending, or

  - for the existence of non-stationarity in the series of government debt and budget balances.

- The government budget constraint, in real terms, not considering revenues from the issuance of money, is

$$B_t - B_{t-1} = r_t B_{t-1} + G_t - R_t \qquad (1)$$

G – government spending without interest payments;
R – government revenue;
B – stock of outstanding government debt;
r – real interest rate.

$$B_t = (1+r_t)B_{t-1} + G_t - R_t \qquad (2)$$

- The difference equation (2) relates the stock of government debt in *t* with the primary budget balance in *t*, plus interest payments stemming from *t-1*.

i) With balanced primary balances, $G_t=R_t$, the growth rate of government debt equals the real interest rate;

ii) With primary deficits, $G_t-R_t>0$, the growth rate of government debt is higher than the real interest rate;

iii) With primary surpluses, $G_t-R_t<0$, the growth rate of government debt is lower than the real interest rate.

- Considering seigniorage revenues,

$$B_t = (1+r_t)B_{t-1} + G_t - R_t - \frac{(M_t - M_{t-1})}{P_t} \qquad (3)$$

M – nominal monetary base;
P – price level.

- Using,

$$Y_t = (1+y_t)Y_{t-1} \qquad (4)$$

Y – real GDP;
y – real GDP growth rate;

- The government budget constraint as a ratio of GDP is

$$\frac{B_t}{Y_t} = \frac{(1+r_t)}{(1+y_t)}\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t}{Y_t} - \frac{R_t}{Y_t} - \frac{M_t - M_{t-1}}{P_t Y_t} \qquad (5)$$

with $b_t = B_t/Y_t$, $g_t = G_t/Y_t$, $\rho_t = R_t/Y_t$,

$$b_t = \frac{(1+r_t)}{(1+y_t)} b_{t-1} + g_t - \rho_t - \frac{M_t - M_{t-1}}{P_t Y_t} \qquad (6)$$

$$M_t = (1+\lambda_t)M_{t-1} \qquad (7)$$

- The government budget constraint (6), includes seigniorage, τ,

$$\tau = \frac{M_t - M_{t-1}}{P_t} \qquad (8)$$

$$\tau = \frac{M_t}{P_t}\frac{\lambda_t}{1+\lambda_t} \qquad (9)$$

τ is the product of real monetary base (the base for the inflation tax) and the inflation tax (growth rate of monetary based divided by $(1+\lambda_t)$),

$$(1+\lambda_t) = (1+y_t)(1+\pi_t).$$

- Seigniorage can also be measured as a % of GDP ($\Delta M/PY$).

- Recalling the money demand equation from the quantity theory of money,

$$\frac{M_t}{P_t} = kY_t \qquad (10)$$

$1/k$ – (constant) velocity circulation of money

- Seigniorage, as a ratio of GDP, τ^*, is derived from (9) and (10)

$$\tau^* = k\frac{\lambda_t}{1+\lambda_t} \qquad (13)$$

$$\tau^{*} = k\frac{\lambda_t}{1+\lambda_t} \qquad (13)$$

- Using (13) in the government budget constraint (6)

$$b_t = \frac{(1+r_t)}{(1+y_t)}b_{t-1} + g_t - \rho_t - k\frac{\lambda_t}{1+\lambda_t} \qquad (14)$$

- The government can sell assets (privatizations) Z ($z = Z/Y$), to finance a budget deficit.

$$b_t = \frac{1+r_t}{1+y_t} b_{t-1} + g_t - \rho_t - k\frac{\lambda_t}{1+\lambda_t} - z_t \qquad (15)$$

- From (15), *ceteris paribus*, the growth rate of the debt-to-GDP ratio will decrease when the **real interest rate is lower than the real economic growth**.

- A primary balance can be seen as sustainable if it provides a constant debt-to-GDP ratio:

$$b_t - b_{t-1} = 0 \qquad (16)$$

- The condition for the stability of the debt-to-GDP ratio can be derived from (15),

$$b_t - b_{t-1} = \frac{1+r_t}{1+y_t} b_{t-1} + g_t - \rho_t - k\frac{\lambda_t}{1+\lambda_t} - z_t - b_{t-1} \qquad (17)$$

$$b_t - b_{t-1} = \frac{r_t - y_t}{1+y_t} b_{t-1} + g_t - \rho_t - k\frac{\lambda_t}{1+\lambda_t} - z_t \qquad (19)$$

$$b_t - b_{t-1} = \frac{r_t - y_t}{1+y_t} b_{t-1} + g_t - \rho_t - k \frac{\lambda_t}{1+\lambda_t} - z_t \qquad (19)$$

The development of the debt-to-GDP ratio depends of:

- the product between the accumulated stock of government debt and the difference between the real interest rate and the real growth rate;

- the primary budget balance;

- seigniorage revenues;

- revenues from selling assets.

- One can recall the unpleasant monetarist arithmetic of Sargent and Wallace (1981)
- Since when the real interest rate is above the real economic growth rate, the government needs to keep sufficient high enough primary surpluses, which may prove difficult to achieve.

- From (16) and (19) the necessary primary budget balance that keeps the debt-to-GDP ratio constant is given by

$$\rho_t - g_t = \frac{r_t - y_t}{1+y_t} b_{t-1} - k \frac{\lambda_t}{1+\lambda_t} - z_t \qquad (20)$$

- From (19), and in nominal terms,

$$b_t - b_{t-1} = \frac{i_t - n_t}{1+n_t} b_{t-1} + \left(g_t - \rho_t - k \frac{\lambda_t}{1+\lambda_t} \right) \qquad (35)$$

- With the Fisher equation for the real interest rate, $r=i-\pi$, and for the nominal economic growth rate, $n=y+\pi$,

$$\Delta b = g - \rho - \lambda m + b(i-n) \qquad (36)$$

$$\Delta b = g - \rho - \lambda m + b(r+\pi-y-\pi) \qquad (37)$$

$$\boxed{\Delta b = g - \rho - \lambda m + b(r-y)} \qquad (38)$$

- From (38), debt follows an explosive path when $r > y$.

- Assuming again $\Delta b=0$, (38) can also be solved for b,

$$b = (\rho - g + \lambda m)/(r - y) \qquad (39)$$

- The government budget constraint (33) can be simplified, omitting the monetary base,

$$\Delta b = g + ib - \rho - \lambda m - bn \qquad (33)$$

$$\boxed{\Delta b_t = d_t - nb_t} \qquad (40)$$

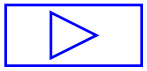
$d = g + ib - \rho$, budget deficit, % of GDP.

- The 3% limit for the deficit in the Stability and Growth Pact of the EU, is an arbitrary value. The underpinnings go back to the beginning of the 1990s.

- A deficit of 3% of GDP is consistent with the long-run convergence of the debt ratio to 60%, assuming a nominal growth rate of 5% (for instance, 3% real growth and 2% inflation).

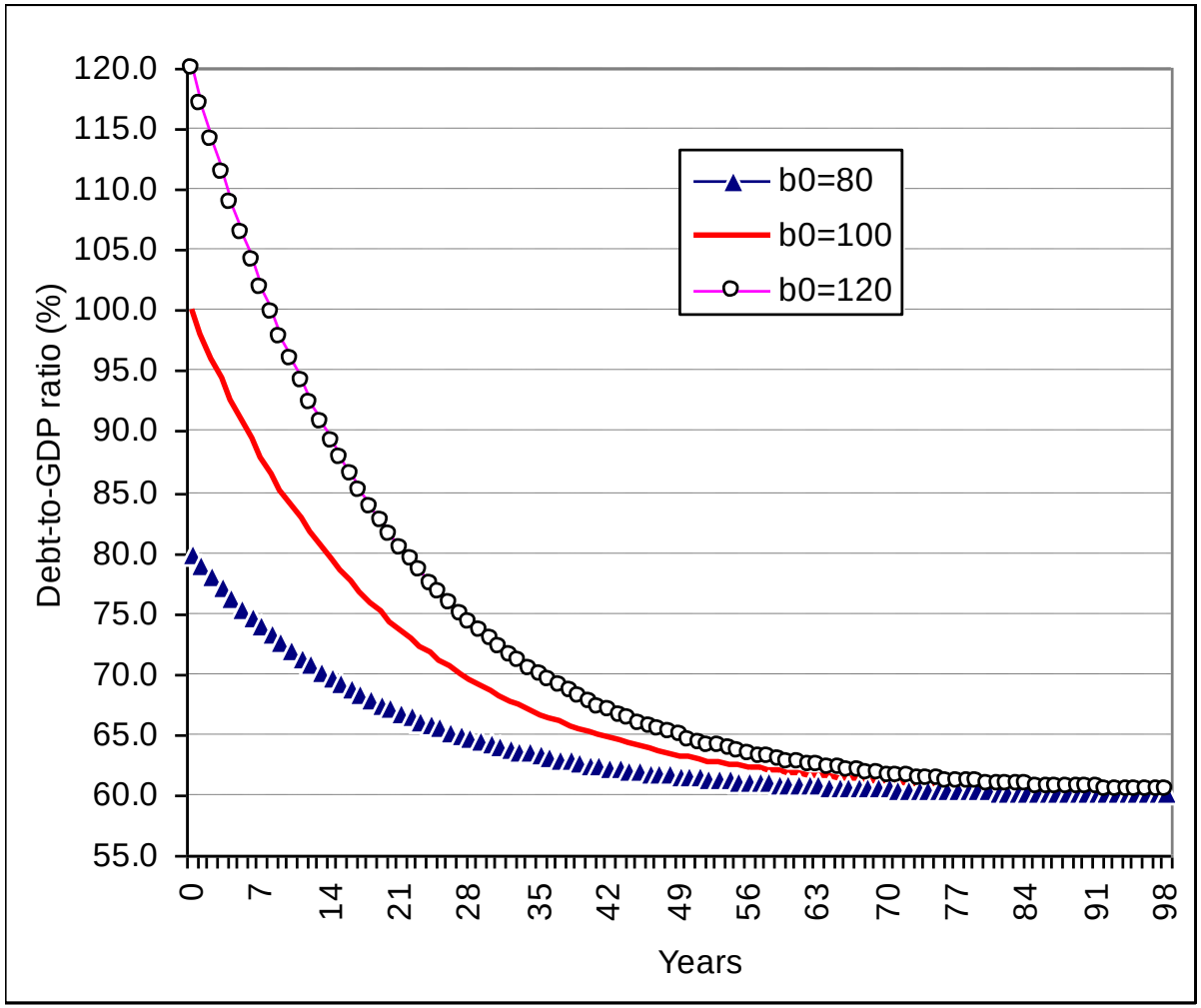
$$\Delta b_t = d_t - n b_t$$

- Trajectories for the debt-to-GDP ratio, for different starting debt ratio values: the ratio converges to $b=d/n$ (0.6=3/5), the limit of the series of the successive debt ratios.

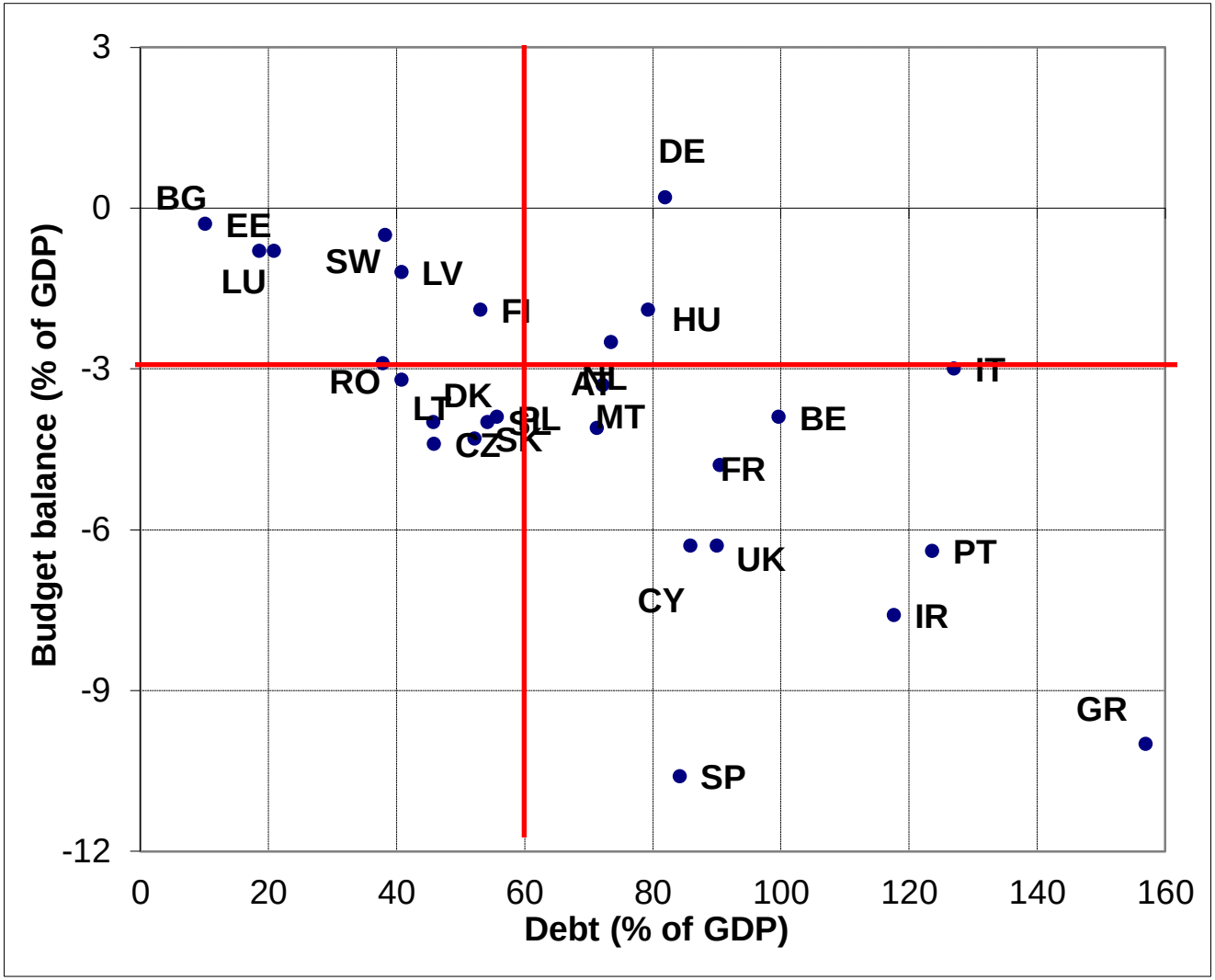


Debt-to-GDP trajectories (n=5%; d=3.5%)

$$\Delta b_t = d_t - n b_{t-1}$$



Budget balance and debt ratios in the EU27, 2012



Source: April 2013, AMECO.

- In the beginning of the 1920s, when writing about the public debt problem faced by France, Keynes (1923) highlighted the need for the French government to conduct a sustainable fiscal policy in order to satisfy its budget constraint, stating that the absence of sustainability would be evident when

"**the State's contractual liabilities (…) have reached an excessive proportion of the national income**" (Keynes, 1923, p. 54).

- In modern terms, the sustainability of public finances is challenged when the government debt-to-GDP ratio reaches an excessive value. Government revenues are not enough to keep on financing the costs associated with the new issuance of public debt or, again in Keynes words, when

"it has become clear that the claims of the bond-holders are more than the tax payers can support" (p. 55). At that point the government will have to take measures that restore the sustainability of fiscal policy, meaning that the State "must come in due course to some compromise between increasing taxation, and diminishing expenditure, and reducing what (…) [it] owe[s]" (p. 59). (see, Afonso and Rault, 2008).

- **Ponzi games**: in the 1920s Charles Ponzi swindled several investors in Boston, making interest payments to previous investors with the money received from new investors. He was arrested when was unable to pay his debts and keep the scheme alive.

- Theoretically, the government can also, to some extent, rollover existing government debt during several years.

- Such approach will be possible if the real interest rate is lower than the real economic growth rate ("honest Ponzi games", as labelled by Buiter, 1985).

- However, in the end, the government has to face its intertemporal budget constraint, and needs to repay its debt, delivering primary budget surpluses.

- Fiscal policy sustainability can be assessed via the so-called present value borrowing constraint (PVBC). In GDP ratios:

$$\frac{B_t}{Y_t} = \frac{(1+r_t)}{(1+y_t)}\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t}{Y_t} - \frac{R_t}{Y_t} \qquad (53)$$

- Assuming the real interest rate to be stationary, with mean *r*, and considering also a constant real GDP growth, the budget constraint is then given by

$$b_{t-1} = \sum_{s=0}^{\infty}\left(\frac{1+y}{1+r}\right)^{(s+1)}\left[\rho_{t+s} - e_{t+s}\right] + \lim_{s\to\infty} b_{t+s}\left(\frac{1+y}{1+r}\right)^{(s+1)} \qquad (55)$$

with $b_t = B_t/Y_t$, $e_t = E_t/Y_t$ and $\rho_t = R_t/Y_t$.

$$e_t = g_t + (r_t - r)b_{t-1}$$

- A sustainable fiscal policy needs to ensure that the present value of the stock of government debt, the 2nd term in the right hand side of (55), goes to zero in infinity,

- Sustainability is more commonly assessed via cointegration analysis between government revenue and spending (Hakkio and Rush (1991)). The implicit hypothesis concerning the real interest rate, with mean *r*, is also stationarity.

-With the no-Ponzi game condition, spending and revenue must be cointegrated of order one for their first differences to be stationary.

- Assuming that spending and revenue are non-stationary variables, and that the first differences are stationary variables, this implies that the original series in levels are I (1).

- If spending and revenue are I(1), these two variables should be cointegrated with cointegration vector (1, -1) for the difference [spending-revenue] to be stationary.

- Assessing the sustainability of the intertemporal government budget constraint involves testing the following cointegration regression (*R*-total revenue, *G*-total spending):

$$R_t = a + bG_t + u_t \qquad (65)$$

- If the null of no cointegration is rejected, then one should accept the alternative hypothesis of cointegration (u_t must be stationary).

- If *G* and *R* are non-stationary variables in levels, $0 < b < 1$ is sufficient for the budget constraint to be obeyed (Hakkio and Rush (1991)).

- When revenues and expenditures are expressed as a % of GDP (or in per capita terms), it is necessary to have $b = 1$ for the trajectory of the debt ratio not to diverge in an infinite horizon.

- Single country cointegration analysis has been the widely used to assess fiscal sustainability (see Afonso (2005) for survey and EU results).

- **Panel data methods have also recently been used to assess fiscal sustainability**, testing for cointegration between government expenditure and revenue (Westerlund and Prohl (2008) for OECD countries, and Afonso and Rault (2008, 2010) for the EU).

- Using a panel approach increases the power that may be brought to the cointegration hypothesis (increased number of observations that results from adding the individual time series).

- Even if there is no single fiscal policy in the EU, panel analysis of the sustainability of public finances is relevant in the context of 27 EU countries seeking to pursue sound fiscal policies within the framework of the Stability and Growth Pact.

- Cross-country dependence can be envisaged either in the run-up to EMU or, for example, via integrated financial markets.
- Cross-country spillovers in government bond markets are to be expected, and interest rates comovements inside the EU have also gradually become more noticeable.

- Testable cointegration relation:

$$R_{it} = \alpha_i + \beta_i G_{it} + u_{it} \quad i=1,\ldots,N;\ t=1,\ldots,T \qquad (66)$$

SUR estimation for EU22 panel (max. time span, 1960-2006)

Country	Coefficients α, β in eq. (66)		t-Statistic	Probability	Country	Coefficients α, β in eq. (66)		t-Statistic	Probability
Austria	α	9.272	12.0	0.000	Latvia	α	12.368	3.0	0.000
	β	0.770	48.0	0.000		β	0.651	5.7	0.000
Belgium	α	9.424	7.4	0.000	Lithuania	α	20.872	12.8	0.000
	β	0.705	28.4	0.000		β	0.362	8.4	0.000
Cyprus	α	-4.104	-0.9	0.331	Luxembourg	α	3.715	3.3	0.001
	β	1.011	9.8	0.000		β	0.942	30.6	0.000
Czech Republic	α	32.669	13.4	0.000	Malta	α	-11.842	-1.2	0.206
	β	0.156	2.9	0.000		β	1.121	5.3	0.000
Denmark	α	6.909	5.4	0.000	Netherlands	α	5.091	6.7	0.000
	β	0.863	34.2	0.000		β	0.845	55.3	0.000
Finland	α	9.449	8.1	0.000	Portugal	α	6.107	9.0	0.000
	β	0.835	32.8	0.000		β	0.713	38.7	0.000
France	α	7.713	13.4	0.000	Romania	α	13.379	4.2	0.000
	β	0.792	64.9	0.000		β	0.602	7.8	0.000
Germany	α	14.342	15.5	0.000	Slovenia	α	0.000	2.1	0.030
	β	0.639	31.0	0.000		β	1.000	8.0	0.000
Greece	α	8.136	10.7	0.000	Spain	α	5.280	10.2	0.000
	β	0.644	36.7	0.000		β	0.780	53.9	0.000
Ireland	α	8.286	5.4	0.000	Sweden	α	23.478	16.5	0.000
	β	0.674	17.9	0.000		β	0.580	22.9	0.000
Italy	α	6.487	4.7	0.000	UK	α	12.944	5.8	0.000
	β	0.698	23.0	0.000		β	0.628	11.6	0.000

SUR, linear estimation after one-step weighting matrix. Balanced system, N=780. (Table 8c, Afonso and Rault, 2008).

Evidence:

- β is always statistically significant, and with the right sign, its magnitude is also below unity.

- The size of the β is quite high in some cases and above, for instance, 0.8, notably for Denmark, Finland, Luxembourg, and the Netherlands (public finances may have been relatively less unsustainable in the past for the abovementioned four countries).

- Lower magnitude of the estimated β coefficient for several countries such as Belgium, Greece, Ireland, Italy, the UK or Sweden (reflects a bigger departure from a one-to-one linkage between expenditures and revenues in the cointegration relationship).

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Background

- Government budget constraint in nominal terms (omitting for simplicity privatization revenues),

$$B_t = (1+i_t)B_{t-1} + G_t - R_t - \left(M_t - M_{t-1}\right) \qquad (21)$$

$$B_t - B_{t-1} = i_t B_{t-1} + G_t - R_t - \left(M_t - M_{t-1}\right) \qquad (22)$$

without subscripts,

$$\Delta B = G - R + iB - \Delta M \qquad (23)$$

- To assess the path of the debt-to-GDP ratio we need to compute the total derivative of *B/Y*:

$$d\left(\frac{B}{Y}\right) = \frac{\partial(B/Y)}{\partial B}dB + \frac{\partial(B/Y)}{\partial Y}dY \qquad (24)$$

$$d\left(\frac{B}{Y}\right) = \frac{1}{Y}dB + \left(-\frac{B}{Y^2}\right)dY \qquad (25)$$

$$d\left(\frac{B}{Y}\right) = \frac{dB}{Y} - \frac{B}{Y}\frac{dY}{Y} \qquad (26)$$

- With a notation similar to (23), for small changes,

$$\Delta\left(\frac{B}{Y}\right) = \frac{\Delta B}{Y} - \frac{B}{Y}\frac{\Delta Y}{Y} \qquad (27)$$

using $b = B/Y$, $n = \Delta Y/Y$, (27) becomes

$$\Delta b = \frac{\Delta B}{Y} - bn \qquad (28)$$

Substituting (23) in (28),

$$\Delta b = \frac{G + iB - R}{Y} - \frac{\Delta M}{Y} - bn \qquad (29)$$

with $g = G/Y$, $\rho = R/Y$,

$$\Delta b = g + ib - \rho - \frac{\Delta M}{Y} - bn \qquad (30)$$

$\lambda = \Delta M/M$, growth rate of the monetary base; $m = M/Y$,

$$\frac{\Delta M}{M}\frac{M}{Y} = \lambda m \qquad (32)$$

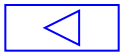
with (32) and (30),

$$\Delta b = g + ib - \rho - \lambda m - bn \qquad (33)$$

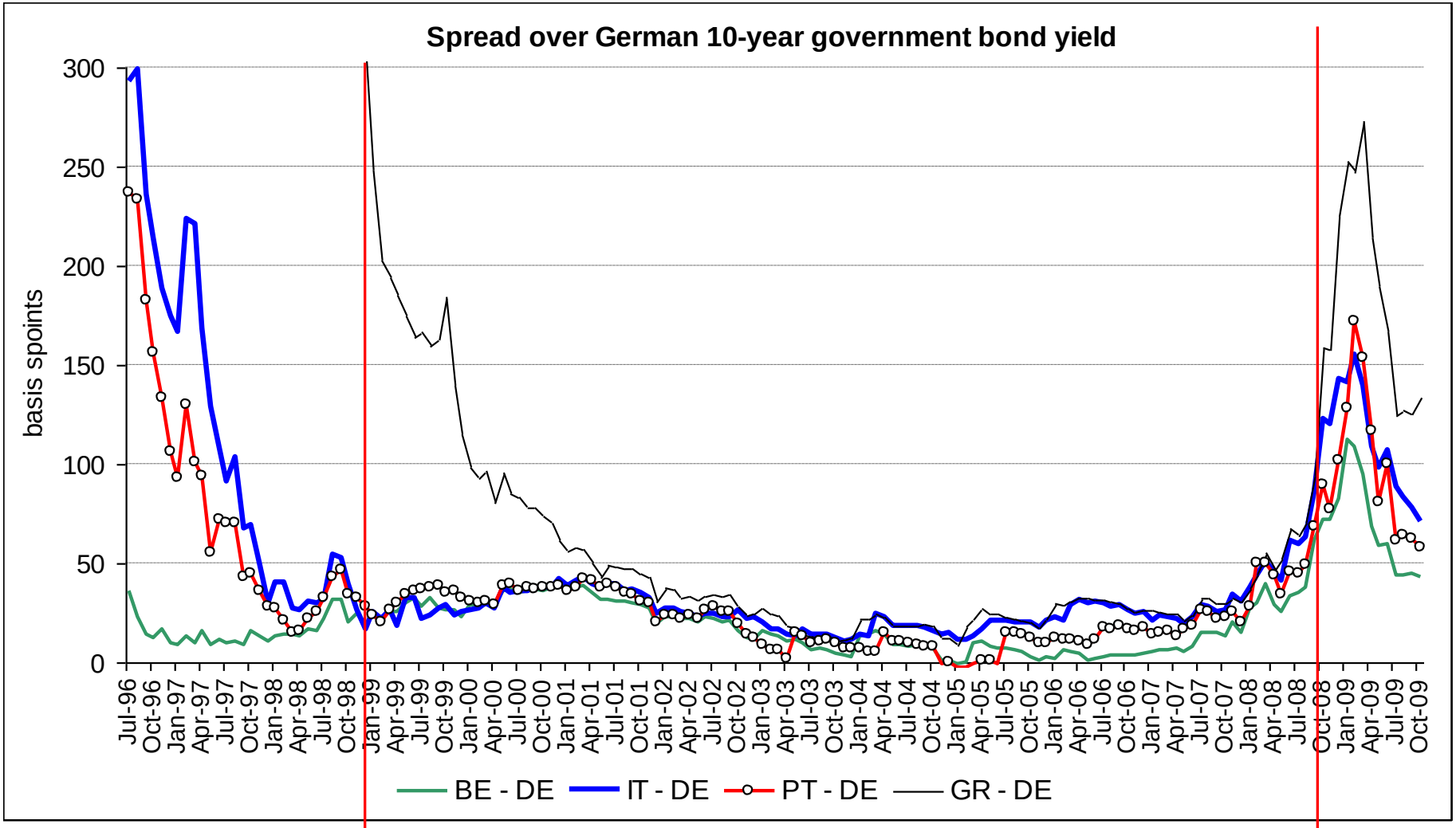
- The long-term government budget constraint implies $\Delta b = 0$,

$$nb = g + ib - \rho - \lambda m \qquad (34)$$

- From (34), in the long-run, the budget deficit is approximately proportional to the level of government debt, and the proportionality factor is the real economic growth rate.



Long-term interest rates 1/

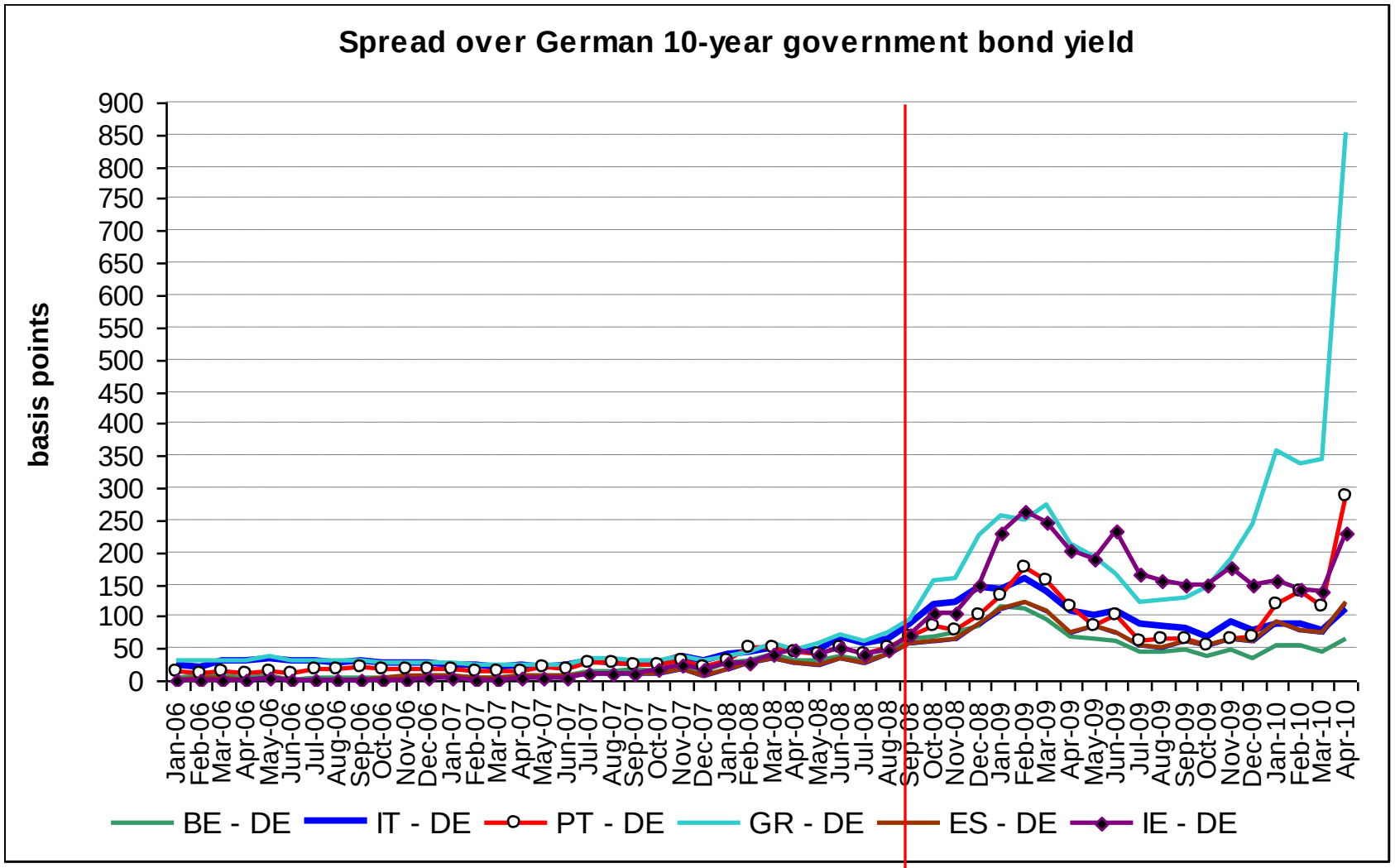


Sources: Reuters.

Jan. 1999, launch of the euro.

Lehman Brothers filed for bankruptcy protection on September 15, 2008.

Long-term interest rates 2/



Sources: Reuters.

Lehman Brothers filed for bankruptcy protection on September 15, 2008.

- Assuming balanced budgets $d_t=0$,

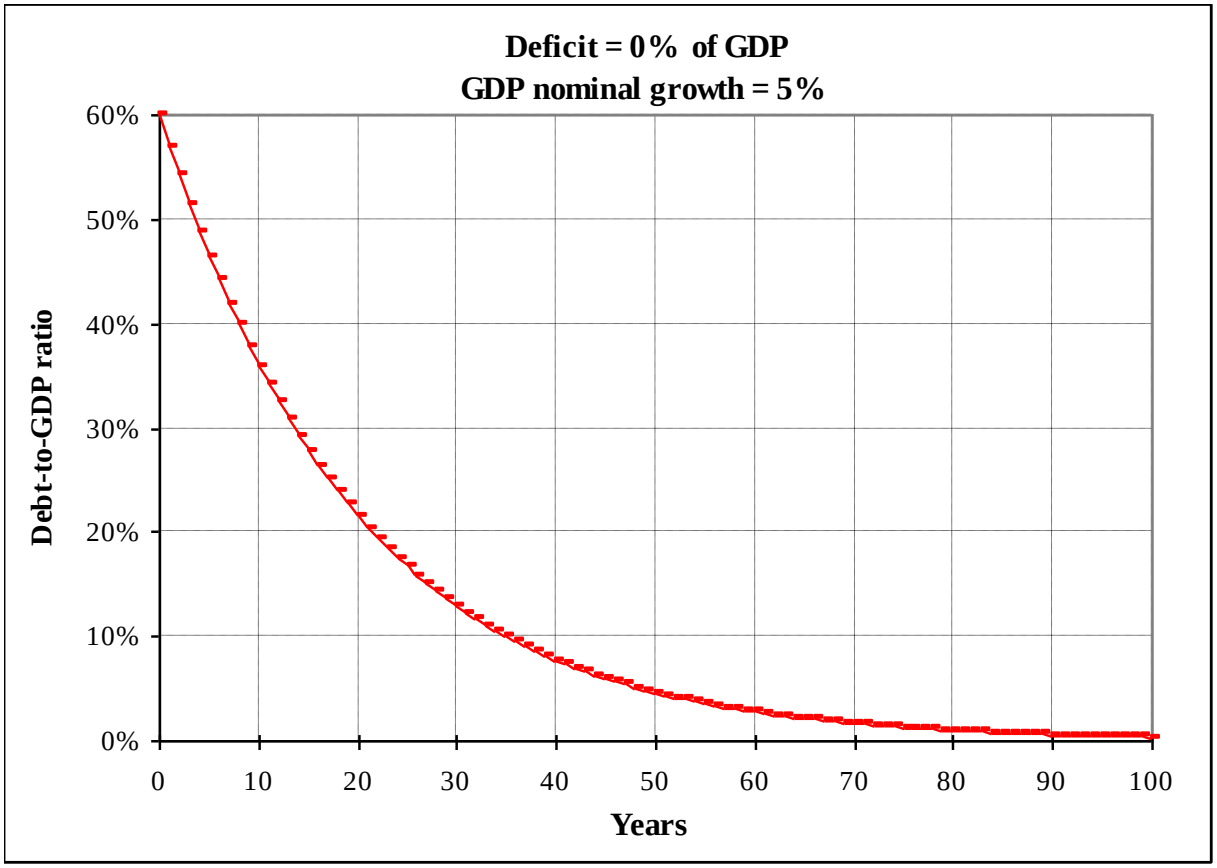
$$\dot{b}_t = -nb_t \qquad (41)$$

- The solution for the differential equation (41) is

$$b_t = Ce^{-nt} \qquad (42)$$

which converges to zero in the long-run since $\lim_{t \to \infty} \left[Ce^{-nt} \right] = 0.$

Simulation of path for the debt-to-GDP ratio



$$\Delta b_t = d_t - nb_{t-1}, \qquad (1)$$

fixing d and n,

$$b_t = d + (1-n)b_{t-1},$$
$$b_{t+1} = d + (1-n)d + (1-n)^2 b_{t-1},$$
$$b_{t+2} = d + (1-n)d + (1-n)^2 d + (1-n)^3 b_{t-1},$$
$$b_{t+k} = d[1 + (1-n) + (1-n)^2 + \ldots + (1-n)^k] + (1-n)^{k+1} b_{t-1}. \qquad (2)$$

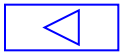
Since (1-n) <1, then $\lim_{k\to\infty}(1-n)^{k+1} b_{t-1} = 0$.

The right-hand side of (2) is d times the content of the square brackets, a geometric series, with the constant multiplier *(1-n)*.

Since its sum is $S = \frac{a_1 - a_n r}{1-r}$, then $S_k = \frac{1-(1-n)^{k+1}}{1-(1-n)}$.

With $\lim_{k\to\infty}(1-n)^{k+1} = 0$, then $\lim_{k\to\infty} S_k = 1/n$, and the content of the square brackets in (2) is *(1/n)*.

Therefore, *b* converges to *d/n*.



Debt-to-GDP trajectories (n=5%; d=3.5%)

$$\Delta b_t = d_t - n b_{t-1}$$